New Approach to Analysis and Design of Smith-Predictor Controllers

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In process control, the Smith-predictor controller introduced by Smith (1957) is a useful technique for deadtime compensation. One main problem with this scheme, however, is that a model of the process is required for output prediction. As in the case of other model-based controllers, it therefore suffers from a sensitivity problem. This has posed a big obstacle to its real-time applications, since in practice, no mathematical model will be a perfect representation of the real process. In realistic situations, there is no guarantee that a Smith-predictor controller will give a better performance over a single-loop controller for the same process even when it has a significant deadtime (Palmor, 1980; Yamanaka and Shimemura, 1987). Therefore, often the use by engineers of a single-loop PID control though deadtime compensation is highly desirable to enhance performance. This has been a persistent problem associated with the practical implementation of the Smith-predictor controller, and thus far there has been no satisfactory solution to this problem.

This article gives a different perspective to the analysis and design of Smith-predictor controllers. An equivalent representation is first presented. It shows that the Smith system contains an inherent phase-compensated element in the feedback loop and, further, the single-loop controller can be viewed as a special mismatched Smith system. Under perturbed conditions, all the uncertainty is concentrated in this element so that its properties are indicative of the achievable closed-loop performance. This provides a unified framework for analysis and design of Smith systems and single-loop controllers, and it clearly shows when we should use the Smith controller and what benefits the controller offers over the single-loop controller. For this assessment a suitable performance measure is formulated based on the frequency response characteristics of the feedback element which depends on the process model used for the output prediction. Process model selection may then be posed as an optimization problem whose solution corresponds to the most desirable performance measure. As a result, we observe that in contrast to intuition a mismatched Smith system may give a better performance over the perfectly matched one. Examples are included for illustration.

The Otto Smith predictor scheme is reviewed. An equivalent representation is given together with an account of the significant features. A general measure for the assessment of the achievable closed-loop performance is formulated. Model identification is considered, the controller design is discussed, and the effects of reduced-order modeling on the closed-loop performance of the Smith-predictor are investigated with several examples.

Smith-Predictor Controller—A Review

The Smith-predictor controller was proposed by Smith (1957) for deadtime compensation and is shown in Figure 1, where $g_p(s) = g_r(s)e^{-Ls}$ and $g_{po}(s) = g_{ro}(s)e^{-L_o s}$ are the process and model, respectively. The closed-loop transfer function between the setpoint and output can be shown to be

$$g_{yr}(s) = \frac{g_c(s)g_p(s)}{1 + g_c(s)\left[g_{ro}(s) - g_{ro}(s)\epsilon^{-sL_o} + g_p(s)\right]} \quad (1)$$

In the case of perfect modeling, i.e., $g_{po}(s) = g_p(s)$, the closed-loop transfer function is

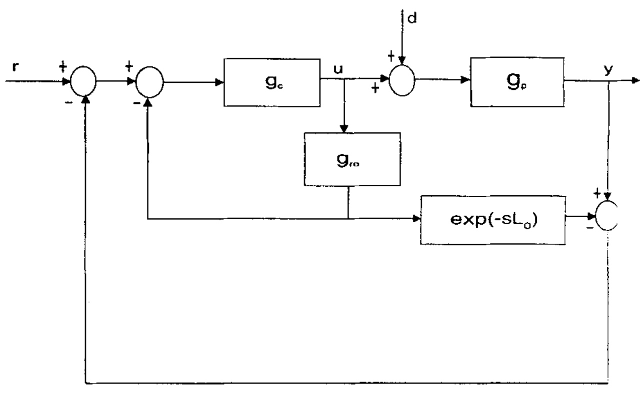

Figure 1. Smith-predictor controller.

$$g_{cl}(s)=\frac{g_c(s)g_r(s)}{1+g_c(s)g_r(s)}e^{-Ls}.$$

This implies that the characteristic equation is free of the delay so that the primary controller $g_c(s)$ can be designed with respect to $g_r(s)$. The achievable performance can thus be greatly improved over a conventional single-loop system without the delay-free output prediction.

Consider, in addition, the presence of a load disturbance shown in Figure 1 as the signal $d(s)$. It is rather straightforward to show that the transfer function between d and y is given by

$$g_{yd}=g_p(1-g_{yr}). \tag{2}$$

For both setpoint tracking and load disturbance rejection, it is thus a common goal for g_{yr} to approach unity for a large frequency range. Traditionally, one thinks that the perfect matching would give the best control results. But this may not be true, as we will show in subsequent sections.

Equivalent Representation

The Smith-predictor structure is in many aspects similar to the internal model control (IMC) structure (Morari and Zafiriou, 1989), which has similar predictor capabilities. In Lee et al. (1995) stability properties of the Smith predictor are inferred from an IMC equivalent representation of the Smith predictor.

Consider yet an equivalent representation of the Smith-predictor controller as shown in Figure 2. The distinction of the Smith system from a single-loop control system is the additional compensator $C(s)$ in the feedback path having the transfer function

$$C(s)=\frac{g_{ro}(s)(1-e^{-sL_o})}{g_r(s)e^{-sL}}+1.$$

The entire feedback element is described by

$$F(s)=C(s)g_p(s). \tag{3}$$

In case of $L_o=0$, it follows that $C(s)=1$ and $F(s)=g_r(s)e^{-sL}$. With this particular model, Figure 2 becomes a single-loop control system. The Smith "compensated" process is now the actual process itself. Thus, we can actually view the single-loop control system as a particular case of a mismatched Smith system with $L_o=0$.

In general, $F(s)$ can thus be regarded as the Smith "compensated" process. Under a perfectly-matched condition, i.e., $g_p(s)=g_{po}(s)$, we have

$$C(s)=e^{sL}, \tag{4}$$

$$F(s)=g_r(s). \tag{5}$$

This indicates that under a perfectly matched condition, the compensator $C(s)$ gives a considerable phase lead in the feedback loop in the form of a deadtime inverse. The total

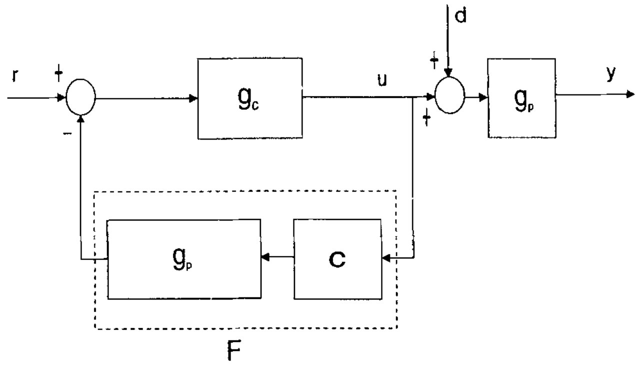

Figure 2. Equivalent representation of the Smith-predictor controller.

phase advance will increase with increasing L. It is this compensator $C(s)$ which neutralizes the deadtime of the process. The Smith "compensated" process $F(s)$ given in Eq. 3 is then the delay-free part of the process; $g_r(s)$ and the controller may thus be designed directly with $g_r(s)$. The concept of inherent phase lead compensation is not new. Astrom (1977) has conjectured that there exists some inherent phase lead compensation in the Smith predictor structure and he verified the point through a specific example.

With this particular representation, every Smith system has an associated "compensated" process $F(s)$ and all the uncertainty is concentrated in this process. The properties of $F(s)$ will thus directly affect the achievable closed-loop performance of the Smith system. For example, if the frequency response $F(j\omega)$ shows the first-order process characteristics, then high gains in the controller $g_c(s)$ are permitted to yield the desirable transfer function g_{yr} for good control performance in terms of setpoint tracking and load disturbance rejection. On the other hand, if the frequency response of $F(s)$ consists of resonant peaks, then the controller $g_c(s)$ has to be considerably detuned for closed-loop stability and poor performance will be expected. The "compensated" process $F(s)$ can thus act as a unifying element in the analysis and design of the Smith-predictor controller. Furthermore, it provides an indication of when to use the Smith system and how to make the best use of it.

Assessment of Achievable Closed-Loop Performance

As described earlier, an assessment of the closed-loop performance can be inferred from the properties of the "compensated" process $F(s)$. Some measures (such as its bandwidth, the relative degree, the relative gain and the relative deadtime) can be used to give a gross indication on the relative ease the process may be controlled, or equivalently the achievable closed-loop performance. A more general measure would be based on the entire frequency response of the process. To give a good achievable closed-loop performance, the process should have low gains, particularly at frequencies where the phase lag/lead is large. Thus, the area enclosed within the Nyquist curve of $F(s)$ could serve as a suitable numerical indicator of the achievable performance. Mathematically, it can be described by

$$J(F) = \int_0^{\theta_{\text{upp}}} W^2(\theta)\bar{F}^2(\theta)d\theta. \quad (6)$$

where $\bar{F}(\theta)$ is the gain of F at the phase lag of θ. W is a phase-weighted specification, analogous to frequency-weighted performance specifications in robust control theory. θ_{upp} is the upper phase range to compute the integral. For general control purposes, a suggested value is $\theta_{\text{upp}} = -\pi$ (Astrom, 1991). The Smith system with "compensated" process $F_1(s)$ can be considered to yield potentially better performance than that with "compensated" process $F_2(s)$ if $J(F_1) < J(F_2)$.

Process Modeling as Optimization Problem

With the formulation of the performance measure, the process model identification problem may be posed as an optimization problem. Unlike traditional identification techniques, we are looking for a model which will yield a more controllable "compensated" process $F(s)$, not necessarily the perfect model. Assuming the actual process is known, it is possible to search for this optimal model $\tilde{g}_{po}(s)$ such that the performance measure J is minimized, i.e.

$$\min_{\forall g_{po}} J(F) = J(F|_{g_{po}=\tilde{g}_{po}}).$$

In general, $\tilde{g}_p(s) \neq g_p(s)$, so that it is possible for a mismatched Smith system to yield a better performance than a perfectly matched one. Further, since we can view the single-loop control system as a special mismatched Smith system with $L_o = 0$, if $\tilde{g}_p(s) = g_{po}(s)|_{L_o=0}$; then the single-loop control system should yield a better closed-loop performance. A general solution to the optimization solution is still a current subject of research. However, at this moment, if given a set Π_o of possible model, we can compute J for the models, and select among them the particular model $\tilde{g}_{po}$ which yields the lowest value of J, i.e., $g_{po} = \tilde{g}_{po} \in \Pi_o$, where

$$\min_{g_{po} \in \Pi_o} J(F) = J(F|_{g_{po}=\tilde{g}_{po}}).$$

This method for process model selection will be illustrated in the examples provided later.

Controller Design

When the optimal model $\tilde{g}_p(s)$ has been obtained, the design of the controller $g_c(s)$ can be carried out based on the Smith "compensated" process $F(s)$. In general, $F(s)$ does not have a well-defined structure so that frequency response design methods would be more general and appropriate. For $g_c(s)$ as PID controllers, off-the-shelves design techniques are available, such as the well-known Ziegler-Nichols tuning rules (Ziegler and Nichols, 1943) and the Astrom-Hagglund tuning rules (Hagglund and Astrom, 1991) which use one point on the Nyquist curve. Robust design methods would detune the controller at frequencies where the frequency response $F(j\omega)$ show resonant peaks (Santacesaria and Scattolini, 1993).

Reduced-Order Modeling and Performance

In real-time applications, no practical systems can be precisely modeled; then all modeling effort is inherently faulty. In process control particularly, reduced-order modeling is often employed where the real high-order process is modeled with a low-order transfer function model with a deadtime (Hang and Chin, 1991; Halevi, 1991). It is then interesting to know the effect of such a process-model mismatch on the performance of the Smith-predictor controller. To this end, some examples are provided for illustration.

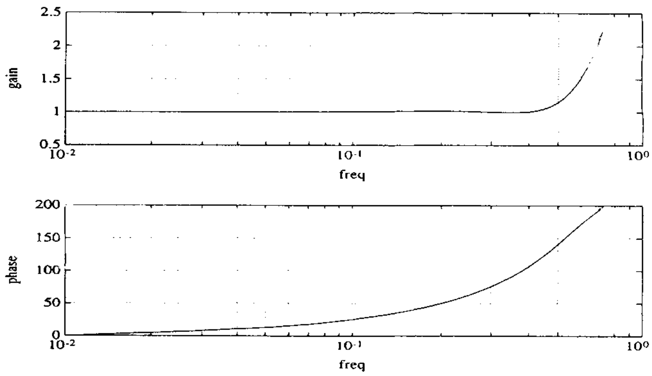

Figure 3. Bode plot of $C(s)$: example 1.

Example 1. Consider a high-order process described by

$$g_p(s) = \frac{1}{(s+1)^{10}}.$$

Assume two models for the process are available: a perfect model $g_{po,1}(s) = g_p(s)$ and a reduced-order model given by

$$g_{po,2}(s) = \frac{1}{(1.94s+1)^3}e^{-4.47s}.$$

Note that the reduced-order model is obtained from the full-order process via a frequency response technique (Hagglund and Astrom, 1991) by matching $g_{po,2}$ to g_p at $\omega = 0$ and $\omega = \omega_u$, where ω_u is the ultimate frequency of g_p.

In the mismatched Smith predictor control system using reduced-order modeling, the function $C(s)$ provides significant phase lead in the ultimate frequency range of the process, as shown by the Bode plot in Figure 3.

The Nyquist plot of the "compensated" process $F(s)$ with the perfect and reduced-order model is shown in Figure 4. In this case, clearly $J(F_2) < J(F_1)$ for any specification W, where F_1 and F_2 are the Smith "compensated" processes associated with $g_{po,1}$ and $g_{po,2}$, respectively.

Finally, the closed-loop responses of the Smith predictor control systems to a setpoint change at $t = 0$ and a 10% load disturbance at $t = 200$ are compared in Figure 5. For a fair comparison, PID controllers are used in both cases, both of which are tuned using the Astrom-Hagglund frequency response method (Hagglund and Astrom, 1991). The Smith system employing reduced-order modeling clearly shows a much better performance compared with the perfectly-matched Smith system.

Example 2. Consider a high-order process with deadtime described by

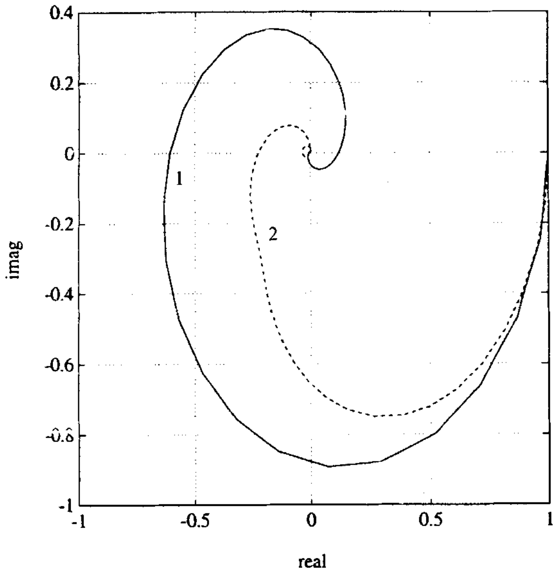

Figure 4. Nyquist plot of $F(s)$ for (1) perfectly matched vs. (2) reduced-order model (example 1).

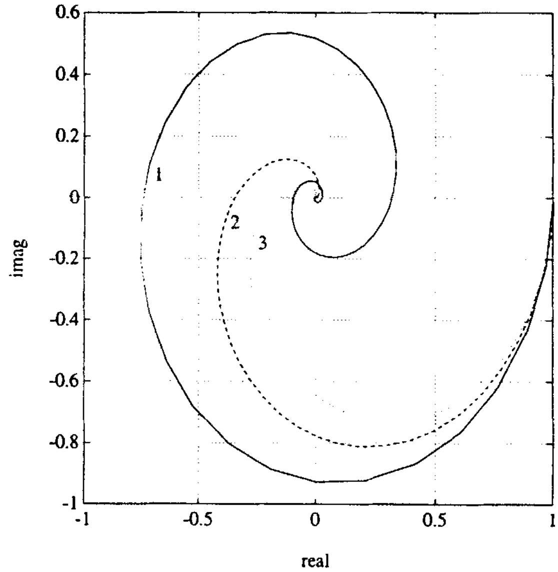

Figure 6. Nyquist plot of $F(s)$ for (1) $L_o = 0$, (2) perfectly matched case, (3) reduced-order model (example 2).

$$g_p(s) = \frac{1}{(s+1)^5} e^{-4s}.$$

Assume three models for the process are available: a mismatched model $g_{po,1}(s)$ with $L_o = 0$, a perfect model $g_{po,2}(s) = g_p(s)$, and a reduced-order model given by

$$g_{po,3}(s) = \frac{1}{(1.32s+1)^3} e^{-5.1s}.$$

The Nyguist plots of the "compensated" process $F(s)$, with the perfect model, the reduced-order model, and with $L_o = 0$ are depicted in Figure 6. $J(F_3) < J(F_2) < J(F_1)$ so that the best performance should be obtained with Smith system employing the reduced-order model. The closed-loop response of the Smith-predictor controller for the three cases are shown in Figure 7. As expected, enhanced improvement is obtained from the Smith system using reduced-order modeling.

Example 3. Consider a high-order nonminimum phase process with deadtime

$$g_p(s) = \frac{-s+1}{(s+1)^5} e^{-2s}.$$

Assume three models for the process are available; a mismatched model $g_{po,1}$ with $L_o = 0$, a perfect model $g_{po,2}(s) = g_p(s)$ and a reduced-order model given by

$$g_{po,3}(s) = \frac{1}{(1.32s+1)^3} e^{-5.1s}.$$

Nyguist plots of the "compensated" process $F(s)$, with the perfect model, the reduced-order model and with $L_o = 0$ are depicted in Figure 8. Again, $J(F_3) < J(F_2) < J(F_1)$ so that the best performance should be obtained with Smith system employing the reduced-order model. The closed-loop response

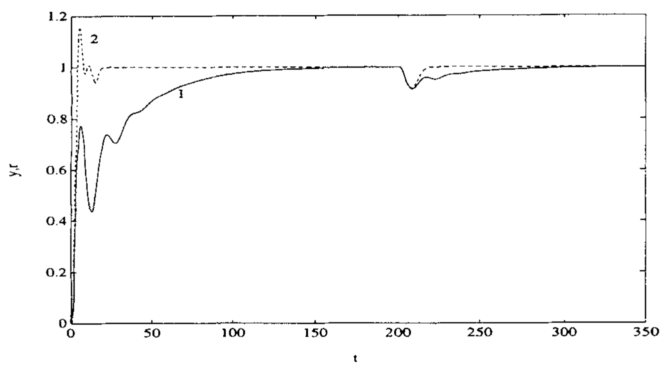

Figure 5. Closed-loop performance: (1) perfectly matched vs. (2) reduced-order model (example 1).

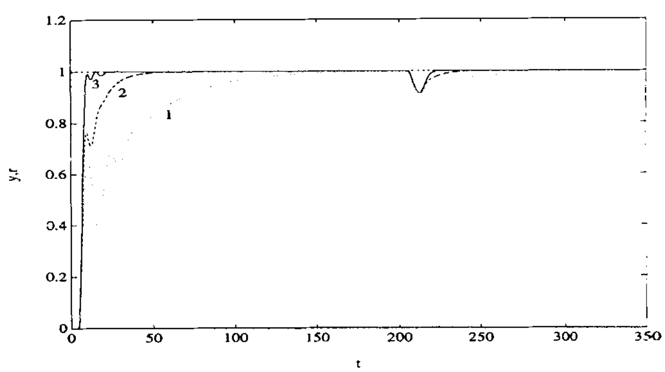

Figure 7. Closed-loop performance: (1) $L_o = 0$, (2) perfectly matched case, (3) reduced-order model (example 2).

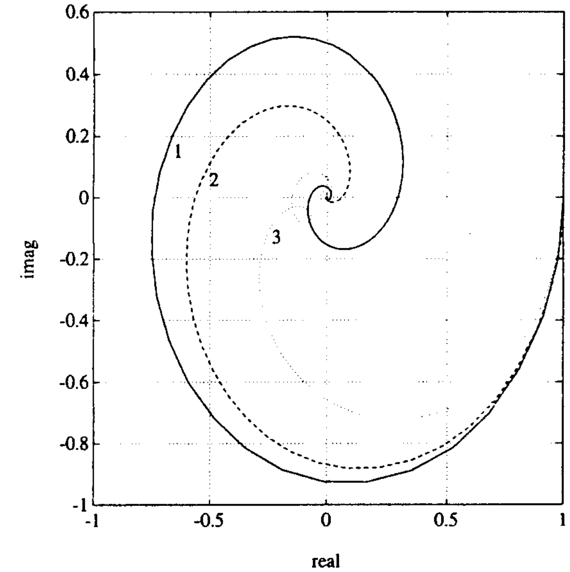

Figure 8. Nyquist plot of $F(s)$ for (1) $L_o = 0$, (2) perfectly matched case, (3) reduced-order model (example 3).

of the Smith-predictor controller for the three cases are shown in Figure 9. Yet again, enhanced improvement is obtained from the Smith system using reduced-order modeling.

Conclusions

This article has presented a new approach to the analysis and design of Smith-predictor controllers using an equivalent representation of the original Smith system. On this platform, mismatched Smith systems can be analyzed in terms of a single mismatched element, and the single-loop control system becomes a special mismatched Smith system. Furthermore, process modeling identification can be posed as an optimization problem. A counterintuitive phenomenon is also illustrated where a mismatched Smith system may yield a better performance over a perfectly-matched one. The author believes that the proposed platform for the analysis and design of Smith-predictor controllers can churn several research opportunities into improving the performance of the well-known control system.

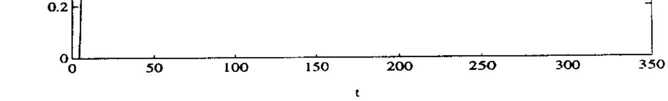

Figure 9. Closed-loop performance: (1) $L_o = 0$, (2) perfectly matched case, (3) reduced-order model (example 3).

Notation

g_{cl} = closed-loop transfer function between r and y
g_p = transfer function of the process
g_{po} = transfer function of the process model
g_r = delay-free part of g_p
g_{ro} = delay-free part of g_{po}
L = deadtime of g_p
L_o = deadtime of g_{po}
r = setpoint
s = Laplace transfer variable
t = time variable
y = process output
Π_o = set of possible g_{po}s

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